1. A machine is purchased for $15,000 and depreciates at the rate of $1,200 per annum. What will be its book value at the end 6 years?

\[ 15,000 + (6)(-1,200) \text{ —note 6 not (6-1) as it's the end of year 6.} \]

2. An air-conditioning unit costs $45,000 and is expected to last 11 years after which it will have a scrap value of $5,000. If it is depreciated by a constant sum each year what is the annual depreciation?

\[ x + (n-1)c = 45,000 - (12-1)c = 5,000 \]

\[ c = \frac{45,000 - 5,000}{12-1} \text{ giving } c = 3,636.36 \]

Check: \[ 45,000 - (12-1)3,636.36 = 5,000 \]

3. A recent graduate wanted to reach a salary of $55,000 after 5 years, with a starting salary of $35,000. What annual fixed increment will she need to achieve her goal?

\[ 35,000 + (5-1)c = 55,000 \]

\[ c = \frac{55,000 - 35,000}{4} = 4,000 \]

4. The total value of a 10 year lease is $1,000,000. If the rental for the first year is $50,000 what annual fixed increment will produce a total amount over the 10 year period.

\[ 1,000,000 = 10[50,000 + 0.5(10-1)c] \]

\[ \frac{1,000,000}{10} = 50,000 + 4.5c \text{ or } 100,000 - 50,000 = 4.5c \]

\[ c = \frac{50,000}{4.5} = 11,111.11 \]

5. An employee receives a total earnings over a 6 year period amounting to $250,000. Salary in the 6th year is $48,000. What is the employee’s starting salary, her salary in the 4th year, and the annual salary increase (assuming a fixed annual increment)?

\[ S = 6[x + 0.5(6-1)c] = 250,000 \]

\[ x + 2.5c = \frac{250,000}{6} = 41,666.67 \]

Year 6: \[ x + (6-1)c = 48,000 \]

The two simultaneous equations to be solved are:
x + 2.5c = $41,666.67
x + 5c = $48,000

From this we get x = $35,333.33 and c = $2,533.33

6. $1,500 is invested for 12 years at 8% per annum compound interest. What will be the amount at the end of the 12th year?

\[ x_k^{n-1} = 1,500(1.08)^{12} = 3,777.26 \]

7. A sum of $500 has been invested for 6 years at a fixed annual interest rate. At the end of the period the total sum is $950. What interest rate was used?

\[ S = 500(1+i)^6 = 950 \]
\[ (1+i)^6 = \frac{950}{500} \]
\[ \text{and } i = \left[ \frac{\frac{950}{500}}{6} \right]^{\frac{1}{6}} - 1 = 0.1129 \text{ or } 11.29\% \]

8. A building owner wishes to accumulate $120,000 for refurbishment in 5 years time. If he can get an interest rate of 9.5% for the 5 year period, what should his initial investment be?

\[ S = \frac{x(k^n - 1)}{k - 1} = \frac{x(1.095^5 - 1)}{1.095 - 1} = 120,000 \]
\[ $x = \left[ \frac{(120,000)}{(1.095^5 - 1)} \right] = 19,852.37 \]

9. The amount of $4,000 is invested for 6 years and 8 months, at 12% per annum, compounded monthly. (i) What is the future value of the sum invested? (ii) If interest is compounded continuously what is the FV?

(i) \[ j = \frac{12\%}{12} = 1\%, \quad n = 6 \times 12 + 8 = 80 \text{ months} \]
\[ S = 4,000(1.01)^{80} = 4,000(2.2167) = 8,866.86 \]

(ii) \[ S_t = 4,000e^{(0.12)(6.6667)} = 4,000(2.22554) = 8,902.16 \]

10. On her twenty first birthday Rebecca received a lump sum of $25,000. If the interest rate was 9% per annum, continuously compounded, for 3.5 years, what initial sum is required to produce the lump sum?

\[ FV = S_t = Pe^{jt}, \text{ rearranging for PV we have } P = S_t e^{-jt} \]
\[ P = \frac{25,000}{e^{(0.09)(3.5)}} = \frac{25,000}{1.370259} = 18,244.72 \]

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*A Spreadsheet Approach to Business Quantitative Methods*
11. What is the present value of $550 paid monthly to a fund earning 8.5% per annum, compounded monthly, for a period of 12 years?

\[
PV = \frac{1 - (1+i)^{-n}}{i} \quad i = \frac{8.5\%}{12} = 0.70833\%, \quad n = 12 \times 12 = 144
\]

\[
PV = 550 \left[ \frac{1 - (1.0070833)^{-144}}{0.0070833} \right] = 550(90.08558) = 49,547.07
\]

12. A building block is sold, by way of vendor finance, for $20,000 on the basis of $5,000 deposit and the balance over 5 years at 10%, payable and adjusted quarterly. The vendor has approached a finance house, who requires a return of 17%, to sell the contract for cash. What is the net return on the sale to the vendor?

Contract price less deposit = $15,000, interest = \(\frac{10\%}{4} = 2.5\%\)

\[
PV = \frac{1 - (1+i)^{-n}}{i} \quad PMT
\]

\[
15,000 = \frac{1 - (1.025)^{-20}}{0.025} \quad PMT = [15.58916] \quad PMT
\]

giving \( PMT = \frac{15,000}{15.58916} = 962.21 \)

The finance house is purchasing the right to $962.21 per quarter for 20 quarters.

The present value of this income stream to the Finance house is obtained as follows. Interest rate = 17%/4 = 4.25%.

\[
PV = \frac{1 - (1.0425)^{-20}}{0.0425} \quad $962.21
\]

\[
= (13.294366)(962.21) = 12,791.97
\]

Deposit \quad $5,000.00
Discounted value of sale of contract \quad $12,791.97
Total cash value of sale \quad $17,791.97

13. Dale is entitled to a fee simple of a property in 7 years with a current market rental of $8,000 per annum, in advance. Using an interest rate of 10%, what is the present value of Dale's entitlement?

\[
PV = \frac{1 - (1+i)^{-n}}{i} (1+i)
\]

\[
PV = 8,000 \left[ \frac{1 - (1.1)^{-7}}{0.1} \right](1.1) = 42,842.08
\]
14. A property has been sold for $245,000 and a deposit of $24,500 was paid on the date of purchase. Under the terms of the contract the balance is paid over a period of 12 years with an interest rate of 15%. If payments are made at the end of the period what is the mortgage constant and the annual payment required to pay off the loan?

Loan $245,000 - $24,500 = $220,500

From (1.23) the mortgage constant is

\[
\frac{1}{A_{n|i}} = \left[ \frac{1 - (1+i)^{-n}}{i} \right]^{-1}
\]

\[
\frac{1 - (1.15)^{-12}}{0.15} = 0.184480776
\]

Payment = \( P = \frac{PV}{A_{n|i}} = $220,500(0.184480776) = $40,678.01 \)

15. Using points of inflection (i.e where \( f'(x) = 0 \)) sketch the following functions.

(i) \( y = f(x) = -2x^2 + 3x \)

First order condition = \( f'(x) = -4x + 3 = 0 \) giving \( x = \frac{3}{4} \)

SOC (second order condition) = \( f''(x) = -4 \)

The SOC is negative, -4, therefore the function is concave.

(ii) \( y = f(x) = 4x^2 + 6 \)

First order condition = \( f'(x) = 8x = 0 \) giving \( x = 0 \)

Second order condition = \( f''(x) = 8 \)

The SOC is positive, 8 > 0, therefore the function is convex.
(iii) \( y = f(x) = x^3 - 6x^2 \)

First order condition = \( f'(x) = 3x^2 - 12x = 0 \)

Solving for \( x \): \( 3x^2 = 12x \) or \( x^2 = 4x \), giving \( x = 4 \)

Second order condition = \( f''(x) = 6x - 12 \)

When \( x > 2 \) the SOC is positive, when \( x < 2 \) then \( f''(x) < 0 \) and when \( x = 2 \), \( f''(x) = 0 \). The function may be either concave, \( f''(x) < 0 \), or convex, \( f''(x) > 0 \), for different values of \( x \).

(iv) \( y = f(x) = \frac{1}{x^2} \), for \( x > 0 \)

FOC: \( f'(x) = -2x^{-3} = -\frac{2}{x^3} = 0 \) \nSOC: \( f''(x) = \frac{6}{x^4} \)
When $x = 0$, or very close to 0, the function $f(x) = \frac{1}{x^2}$ is undefined.
For values of $x > 0$, $f(x)$ decreases and continues to decrease indefinitely. The function is convex for values of $x > 0$ since $f''(x) > 0$ for all $x > 0$.

16. Assume that the demand for a valuer's services is given by the equation:

$$D_V = 6500 - 12x$$

where $x$ represents the valuer's fee. The valuer operates from a small office for which he pays rent and he also uses a motor vehicle, both of which may be classified as fixed costs.

(i) Given that his fixed costs are $3,500, there are no variable costs, and he requires a net profit of $65,000 from this service what average fee should he charge?

(ii) Given his revenue and cost structure what fee will maximise his profit? Using the profit function, demonstrate this with the aid of a diagram. Use a starting fee of $30 and increment by $30 to a fee of $600.

(iii) Using calculus find the optimum point of the profit function and show that this point is a maximum.

(iv) What will the demand for his service be at this price and what is his profit? Is this demand sustainable?

(i) Profit = revenue – cost

$$\text{Profit} = (\text{price})(\text{demand}) - \text{fixed costs}$$

$$\$65,000 = 6,500x - 12x^2 - \$3,500$$

Re-arranging we have a polynomial of degree 2 (a quadratic):

$$12x^2 - 6,500x + 68,500 = 0$$

To find the roots of a quadratic equation we can use the expression:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
\[
-\frac{(-6500) \pm \sqrt{(-6500)^2 - 4(12)(68500)}}{2(12)}
\]

The two roots are: \( x = 530.91 \) and \( x = -10.75 \).
Thus the valuer should charge a fee of approximately $531.

(ii) | \( x \) | **Profit** | \( - \) | **Profit** |
--- | --- | --- | --- | ---
30 | 180,700 | 330 | 834,700 |
60 | 343,300 | 360 | 781,300 |
90 | 484,300 | 390 | 706,300 |
120 | 603,700 | 420 | 609,700 |
150 | 701,500 | 450 | 491,500 |
180 | 777,700 | 480 | 351,700 |
210 | 832,300 | 510 | 190,300 |
240 | 865,300 | 540 | 7,300 |
270 | **876,700** | 570 | (197,300) |
300 | 866,500 | 600 | (423,500) |

(iii) Profit = revenue - cost  
\( f(x) = 6,500x - 12x^2 - 3,500 \)  
\( f'(x) = 6,500 - 24x = 0 \) \( \text{note } f''(x) = -24, \text{ indicating a maximum} \)  
\( x = 271 \text{ (approx.)} \)

(iv) Demand = 6,500 - 12x = 6,500 - 12(271) = 3,248  
Profit = 6,500x - 12x^2 - 3,500  
\( = 6500(271) - 12(271)^2 - 3500 = $876,708 \)
17. A mortgage provider has determined that loan applications, LA, may be described by the equation:

\[ LA = aI + bP \]

where I is the mortgage interest rate and P is promotional expenditure in thousands of dollars. From past records the firm’s data analyst found that when the interest rate was 10% and promotional expenditure was $10,000 the demand for loans was 870. For an interest rate of 12% and promotional expenditure of $12,000 loan applications were 800.

(i) The information may be expressed in the form of two simultaneous equations:

\[ 600 = a(0.10) + b(10) \]  
\[ 750 = a(0.11) + b(12) \]

(ii) Plot these equations and identify the solution, if it exists.

(iii) Solve for the solution algebraically.

Multiplying the first equation by 1.1 and subtracting the first equation from the second produces:

\[ -[660 = a(1.1) + b(11)] \]
\[ 750 = a(1.1) + b(12) \]

\[ b = 90 \] and by substitution \( a = -3,000 \)

The negative value for \( a \) indicates that as the interest rate increases the number of loans demanded is reduced. The positive value for \( b \) suggests...
that promotional expenditure has a beneficial effect, loans increase with promotion. The firm may use this information to determine a loan applications for a range of interest rates and varying amounts of promotional expenditure.

(iv) What would loan applications be if the interest rate was 9% and promotional expenditure $15,000?

\[ \text{LA} = -3,000I + 90P \]

\[ \text{LA} = -3,000(0.09) + 90(15) = 1,080 \]

18. Mr. Andrew Wise is considering a new job. The starting salary is $29,000 with a guaranteed annual raise of $1,000 per year. He also knows that if he works hard his rise will be increased by $200 per year.

(a) What is the guaranteed rate of change of Mr. Wise's salary with respect to time (in years)?

(b) What is the rate of change of his rise with respect to time if he works hard?

(c) What is the derivative of his guaranteed salary with respect to time?

(d) What is the derivative of his salary with respect to time if he works hard?

(e) Write his salary, S, as a function of time \( t = 1, 2, 3, \ldots \) if he works hard. Do the same for the derivative \( \frac{dS}{dt} \) of his salary with respect to time.